

# Microscopic Description of Isospin Mixing Pairing Correlations in the Framework of an Algebraic $\text{Sp}(4)$ Model

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## Abstract

We explore isospin mixing beyond that due to the Coulomb interaction in the framework of an exactly solvable microscopic  $\text{sp}(4)$  algebraic approach. Specifically, we focus on the isospin non-conserving part of the *pure* nuclear pairing interaction. The outcome of this study shows the significance of the pairing charge dependence and its role in mixing isospin multiplets of pairing-governed isobaric analog  $0^+$  states in light and medium mass nuclei, especially in nuclei with equal numbers of protons and neutrons. The model reveals possible, but still extremely weak, non-analog  $\beta$ -decay transitions and estimates their relative strengths within a shell closure.

## 1 Introduction

A major simplification of the pairing problem is achieved if one assumes a charge independent nuclear interaction (the proton-proton  $pp$  interaction and the neutron-neutron  $nn$  interaction are equal to the isospin  $T = 1$  proton-neutron  $pn$  interaction). The latter comprises a quite reasonable approximation and consequently most of isovector (isospin 1) pairing studies have been carried under the assumption of isospin invariance. However, “the problem of broken symmetry is one of general significance

in nuclear (e.g. [1]) and elementary particle (e.g. [2]) physics” [3] (Vol. I, p.37), which has been of long-standing interest [4, 5, 6, 7, 3, 8, 9, 10] and may be associated with novel and interesting physics [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. Experimental results clearly reveal the existence of isospin mixing [23, 24]. An increase in isospin mixing towards medium mass nuclei has been detected in novel high-precision experiments [25, 26, 27, 28, 1], which continue to push the exploration of unstable nuclei with the advent of advanced radioactive beam facilities.

The isospin symmetry in nuclei is slightly violated by the electromagnetic interaction, mainly the Coulomb repulsion between nucleons [3, 6, 11]. Another source of mixing probability is the isospin non-conserving part of the nuclear Hamiltonian, which includes effects due to the proton-neutron mass difference ( $\Delta m/m = 1.4 \times 10^{-3}$ ) and small charge dependent components in the strong nucleonic interaction that appear to be associated with the electromagnetic structure of the nucleons [6]. An analysis of the  $^1S$  scattering in the  $pn$  system and the low-energy  $pp$  scattering lead to the estimate that the pure nuclear interaction between protons and neutrons ( $V_{pn}^{T=1}$ ) in  $T = 1$  states are more attractive than the force between the protons ( $V_{pp}$ ) by 2%,  $|V_{pn}^{T=1} - V_{pp}|/V_{pp} \sim 2\%$  [29]. Furthermore, after the

Coulomb energy is taken into account the discrepancy in the isobaric-multiplet energies is bigger for the seniority zero levels as compared to higher-seniority states indicating the presence of a short range charge dependent interaction [9]. Indeed, the  $J = 0$  pairing correlations have been recently shown to have an overwhelming dominance in the isotensor energy difference within isobaric multiplets [30], which manifests itself in the charge dependent  $T = 2$  nature of the pairing interaction.

The aforementioned findings set the need for a charge dependent microscopic description of  $J = 0$  pairing correlations. For this reason we employ a simple but powerful group-theoretical model [31, 32], which is based on the  $\mathfrak{sp}(4)$  algebra (isomorphic to  $\mathfrak{so}(5)$  [33, 34, 35]). A comparison with experimental data demonstrates that the  $\text{Sp}(4)$  model provides a reasonable description of the pairing-governed isobaric analog  $0^+$  states<sup>1</sup> in light and medium mass nuclei, where protons and neutrons occupy the same shell [31, 36, 32]. The  $\text{Sp}(4)$  model is precisely suitable for the microscopic modeling of the pairing interaction and its isospin violation in isobaric analog  $0^+$  states because it naturally extends the isospin invariant nuclear interaction to incorporate isospin non-conserving forces, while it retains the  $\text{Sp}(4)$  dynamical symmetry of the Hamiltonian. Hence it provides a straightforward scheme for estimating the significance of the isospin mixing due to pairing correlations without the need for carrying out large-dimensional matrix diagonalizations. Strong isospin breaking in pair formation, if found, implies a significant presence of isospin admixture among the seniority-zero isobaric analog  $0^+$  states including  $0^+$  ground states. This in turn will affect the predictive power of precise studies of superallowed  $0^+ \rightarrow 0^+$  Fermi  $\beta$ -decay transitions. Laboratory and theoretical investigations of such transitions provide reliable tests of isospin mixing (see [37] for a review). In addition, the results of examinations on isospin mixing are essential to another challenging problem; namely, when compared

<sup>1</sup>The lowest among these states include ground states for even-even nuclei and only some ( $N \approx Z$ ) odd-odd nuclei, as well as, for example, low-lying  $0^+$  states in odd-odd nuclei that have the same isospin as the ground state of a semi-magic even-even isobaric neighbor with fully-paired protons (or neutrons).

to the decay rate for purely leptonic muon decay, the estimate for the nuclear Fermi  $\beta$  decay rate furnishes a precise test of the unitary condition of the Cabbibo-Kobayashi-Maskawa matrix [38] under the assumption of the three-generation standard particle model (for a review of this subject, see [39]).

## 2 Isospin Mixing of the Isobaric Analog $0^+$ States

The  $\text{Sp}(4)$  model reflects the symplectic dynamical symmetry of isobaric analog  $0^+$  states [32] determined by the strong nuclear interaction. The weaker Coulomb interaction breaks this symmetry and significantly complicates the nuclear pairing problem. This is why, in our investigation we adopt a sophisticated phenomenological Coulomb correction to the experimental energies such that a nuclear system can be regarded as if there is no Coulomb interaction between its constituents. The *Coulomb corrected* experimental energy,  $E_{\text{exp}}$ , for given valence protons  $N_{+1}$  and neutrons  $N_{-1}$  is adjusted to be

$$E_{\text{exp}}(N_{+1}, N_{-1}) = E_{\text{exp}}^C(N_{+1}, N_{-1}) - E_{\text{exp}}^C(0, 0) + V_{\text{Coul}}(N_{+1}, N_{-1}), \quad (1)$$

where<sup>2</sup>  $E_{\text{exp}}^C$  is the total measured energy including the Coulomb energy [40, 41],  $E_{\text{exp}}^C(0, 0)$  is the binding energy of the core, and  $V_{\text{Coul}}(N_{+1}, N_{-1})$  is the Coulomb correction for a nucleus with mass  $A$  and  $Z$  protons taken relative to the core  $V_{\text{Coul}}(N_{+1}, N_{-1}) = V_{\text{Coul}}(A, Z) - V_{\text{Coul}}(A_{\text{core}}, Z_{\text{core}})$ . The recursion formula for the  $V_{\text{Coul}}(A, Z)$  Coulomb energy is derived in [42] with the use of the Pape and Antony formula [43]

$$V_{\text{Coul}}(A, Z) = \begin{cases} V_{\text{Coul}}(A, Z-1) + 1.44 \frac{(Z-1/2)}{A^{1/3}} - 1.02 & Z > Z_s \\ V_{\text{Coul}}(A, Z+1) - 1.44 \frac{(Z+1/2)}{A^{1/3}} + 1.02 & Z < Z_s, \end{cases} \quad (2)$$

where  $Z_s = A/2$  for  $A$  even or  $Z_s = (A+1)/2$  for  $A$  odd. When  $Z = Z_s$  the Coulomb potential is given

<sup>2</sup>To avoid confusion we mention that in (1) the energies are assumed positive for bound states;  $V_{\text{Coul}}$  is also defined positive.

by

$$V_{Coul}(A, Z_s) = \begin{cases} 0.162Z_s^2 + 0.95Z_s - 18.25 & Z_s \leq 20 \\ 0.125Z_s^2 + 2.35Z_s - 31.53 & Z_s > 20. \end{cases} \quad (3)$$

The Coulomb corrected energies (1) should reflect solely the nuclear properties of the many-nucleon systems.

Assuming charge independence of the nuclear force, the general isoscalar Hamiltonian with Sp(4) dynamical symmetry, which consists of one- and two-body terms and conserves the number of particles, can be expressed through the Sp(4) group generators,

$$H_0 = -G \sum_{i=-1}^1 \hat{A}_i^\dagger \hat{A}_i - \frac{E}{2\Omega} (\hat{T}^2 - \frac{3\hat{N}}{4}) - C \frac{\hat{N}(\hat{N}-1)}{2} - \epsilon \hat{N}, \quad (4)$$

where  $\hat{T}^2 = \Omega\{\hat{T}_+, \hat{T}_-\} + \hat{T}_0^2$  and  $2\Omega$  is the shell dimension for a given nucleon type. The generators  $\hat{T}_\pm$  and  $\hat{T}_0$  are the valence isospin operators,  $\hat{A}_{0,+1,-1}^{(\dagger)}$  create (annihilate) respectively a proton-neutron ( $pn$ ) pair, a proton-proton ( $pp$ ) pair or a neutron-neutron ( $nn$ ) pair of total angular momentum  $J^\pi = 0^+$  and isospin  $T = 1$ , and  $\hat{N} = \hat{N}_{+1} + \hat{N}_{-1}$  is the total number of valence particles with an eigenvalue  $n$ . The  $G, E$  and  $C$  are interaction strength parameters and  $\epsilon > 0$  is the Fermi level energy (see Table I in [32] for estimates). The isospin conserving Hamiltonian (4) includes an isovector ( $T = 1$ ) pairing interaction ( $G \geq 0$  for attraction) and a diagonal isoscalar ( $T = 0$ ) force, which is related to a symmetry term ( $E$ ). The two-body model interaction includes proton-neutron and like-particle pairing plus symmetry terms and contains a non-negligible implicit portion of the quadrupole-quadrupole interaction [44]. Moreover, the Sp(4) model interaction itself, which relates to the whole energy spectrum rather than to a single  $J^\pi = 0^+$   $T = 1$  state, was found to be quite strongly correlated (0.85) with the realistic CD-Bonn+3terms interaction [45] in the  $T = 1$  channel and with an overall correlation of 0.76 with the realistic GXPF1 interaction [46] for the  $1f_{7/2}$  orbit [44]. In short, the relatively simple Sp(4) model seems to be a reasonable approximation that reproduces especially that part of the interaction that is responsible for shaping pairing-governed isobaric analog  $0^+$  states.

Charge dependent but charge symmetric<sup>3</sup> nucleon-nucleon interaction ( $V_{pp} = V_{nn} \neq V_{pn}$ ) brings into the nuclear Hamiltonian a small isotensor component (with zero third isospin projection so that the Hamiltonian commutes with  $T_0$ ). This is achieved in the framework of the Sp(4) model by introducing the two additional terms,

$$H_{IM} = -F \hat{A}_0^\dagger \hat{A}_0, \quad H_{split} = -D(\hat{T}_0^2 - \frac{\hat{N}}{4}), \quad (5)$$

to the isospin invariant model Hamiltonian (4) in a way that the Hamiltonian

$$H = H_0 + H_{IM} + H_{split} \quad (6)$$

possesses Sp(4) dynamical symmetry. The interaction strength parameters  $F$  and  $D$  (5) determined in an optimum fit over a significant number of nuclei (total of 149) [31] yield non-zero values (see Table I in [32] for estimates). As expected from observations, for the  $1d_{3/2}$  level the interaction strengths of all  $pn$ ,  $pp$  and  $nn$  pairing are found to be almost equal ( $T$  is a good quantum number),  $F/\Omega = 0.007$ , and they differ for the  $1f_{7/2}$  and for the  $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$  shells, with the  $pn$  isovector strength being more attractive,  $F > 0$ . The full Hamiltonian (6) yields quantitative results that are superior than the ones with  $F = 0$  and  $D = 0$ ; for example, in the case of the  $1f_{7/2}$  level the variance between the model and experimental energies of the lowest isobaric analog  $0^+$  states increases by 85% when the  $D$  and  $F$  interactions are turned off. For the present investigation the parameters in (4) along with  $F$  and  $D$  (5) are not varied as their values were fixed to be physically valid and to yield reasonable energy [31, 32] and fine structure [36] reproduction for light and medium mass nuclei with valence protons and neutrons occupying the same shell. For these nuclei in the mass range  $32 \leq A \leq 100$ , the pairing-governed isobaric analog  $0^+$  states are well described, but still approximately, by the eigenvectors of the effective Hamiltonian (6) in a basis of fully-paired ( $pp$ ,  $pn$  and  $nn$   $T = 1$  pairs)  $0^+$  states (Table 1).

<sup>3</sup>The charge asymmetry between the  $pp$  and  $nn$  interactions is found to be small; namely, less than 1% [47].

Table 1: Classification scheme of even- $A$  nuclei in the  $1f_{7/2}$  shell. The shape of the table is symmetric with respect to the sign of  $T_0$  and  $n - 2\Omega$ . The operators shown in brackets (and their Hermitian conjugates) generate transitions in the action space spanned by the  $|n, T, T_0\rangle$  isospin eigenstates that are linear combinations of the fully-paired basis states.

$n$	Isospin projection $T_0$					
	2	1	0	-1	-2	-3
0			${}^{40}_{20}\text{Ca}_{20}$ $ 0, 0, 0\rangle$			
2		${}^{42}_{22}\text{Ti}_{20}$ $ 2, 1, 1\rangle$	${}^{42}_{21}\text{Sc}_{21}$ $ 2, 1, 0\rangle$	${}^{42}_{20}\text{Ca}_{22}$ $ 2, 1, -1\rangle$		
4	$\swarrow$	${}^{44}_{23}\text{V}_{21}$ $ 4, 2, 1\rangle$	${}^{44}_{22}\text{Ti}_{22}$ $ 4, 2, 0\rangle$	${}^{44}_{21}\text{Sc}_{23}$ $ 4, 2, -1\rangle$	${}^{44}_{20}\text{Ca}_{24}$ $ 4, 2, -2\rangle$	
6	$\dots$	${}^{46}_{24}\text{Cr}_{22}$ $ 6, 3, 1\rangle$ $ 6, 1, 1\rangle$	${}^{46}_{23}\text{V}_{23}$ $ 6, 3, 0\rangle$ $ 6, 1, 0\rangle$	${}^{46}_{22}\text{Ti}_{24}$ $ 6, 3, -1\rangle$ $ 6, 1, -1\rangle$	${}^{46}_{21}\text{Sc}_{25}$ $ 6, 3, -2\rangle$	$\searrow (A^\dagger_{-1})$
8	$\leftarrow (T_+)$	${}^{48}_{25}\text{Mn}_{23}$ $ 8, 4, 1\rangle$ $ 8, 2, 1\rangle$	${}^{48}_{24}\text{Cr}_{24}$ $ 8, 4, 0\rangle$ $ 8, 2, 0\rangle$ $ 8, 0, 0\rangle$	${}^{48}_{23}\text{V}_{25}$ $ 8, 4, -1\rangle$ $ 8, 2, -1\rangle$	${}^{48}_{22}\text{Ti}_{26}$ $ 8, 4, -2\rangle$ $ 8, 2, -2\rangle$	$\dots$
10	$\vdots$	$\vdots$	$\downarrow (A^\dagger_0)$	$\vdots$	$\vdots$	$\swarrow (A^\dagger_{+1})$

While the second interaction ( $H_{\text{split}}$ ) in (5) takes into account only the splitting of the isobaric analog energies, the first correction induces small isospin mixing (IM). The isospin mixing interaction (5) does not account for the entire interaction that mixes states of same angular momentum and parity but different isospin values. It only describes a possible  $\Delta T = 2$  mixing between isobaric analog  $0^+$  states due to a pure nuclear pairing interaction. While the extent of such isospin admixing is expected to be smaller than the total mixing due to isospin non-conserving terms [8, 12, 13, 15, 37], it may influence precise model calculations depending on the importance of the charge dependence in pairing correlations.

The question regarding how strong individual isospin non-conserving nuclear interactions are [such as (5)] still remains open – there are no sharp answers at the present level of experimental results and microscopic theoretical interpretations. It is only their overall contribution that is revealed by the free

nucleon-nucleon data [29] to be slightly (by 2%) more attractive in the  $pn$   $T = 1$  system than the  $pp$  one. Within the framework of the  $\text{Sp}(4)$  model, the charge dependence of the pure nuclear interaction can be estimated through the comparison of the  $T_0 = 0$  two-body model interaction [(6) with  $\varepsilon = 0$ ] relative to the  $T_0 = 1$  in the  $T = 1$  multiplets, which, for example in the  $1f_{7/2}$  level, is on average  $\sim 2.5\%$ . In addition, the  $\text{Sp}(4)$  model reproduces reasonably well the  $c$ -coefficient in the well-known isobaric multiplet mass equation [4, 10, 48]

$$a + bT_0 + cT_0^2, \quad (7)$$

for the binding energies of isobaric analogs (of the same mass number  $A$ , isospin  $T$ , angular momentum  $J$ , etc.), where the coefficient  $c$  ( $b$ ) depends on the isotensor (isovector) component of the nuclear interaction [i.e., of rank 2 (1) with respect to isospin ‘rotations’]. The requirement that the coefficients of (7) are well reproduced is essential for the isospin non-conserving models [8, 12, 30], which has been

achieved in [8] by increasing (approximately by 2%) of all the  $T = 1$   $pn$  matrix elements relative to the  $nn$  ones and which has lead to a conclusion in [30] that the isotensor nature of the nuclear interaction is dominated by a  $J = 0$  pairing term. In agreement with experiment, the  $c$ -coefficients in the  $\text{Sp}(4)$  model were found to be negative and very close to zero for  $T = 1$  multiplets in the  $1f_{7/2}$  shell. Their average relative to the binding energy of the valence nucleons differs from the corresponding experimental value by only 0.3%. These estimations do not aim to confirm the charge-dependence, which is very difficult at this level of accuracy compared to the broad energy range considered in the model for nuclei with masses  $32 \leq A \leq 100$ . Nonetheless, it reflects the fingerprints of the experimental data in the properties of the model interaction (6).

## 2.1 Non-Analog $\beta$ -Decay Transitions

For a superallowed Fermi  $\beta$ -decay transition ( $0^+ \rightarrow 0^+$ ) the  $ft$  comparative lifetime is nucleus-independent according to the conserved-vector-current (CVC) hypothesis and given by

$$ft = \frac{K}{G_V^2 |M_F|^2}, \quad K = 2\pi^3 \hbar \ln 2 \frac{(\hbar c)^6}{(m_e c^2)^5}, \quad (8)$$

where  $K/(\hbar c)^6 = 8.120270(12) \times 10^{-7} \text{ GeV}^{-4} \text{ s}$  ( $m_e$  is the mass of the electron) and  $G_V$  is the vector coupling constant for nuclear  $\beta$  decay (see for example [12]).  $M_F$  is the Fermi matrix element  $\langle F | \sqrt{2\Omega} T_{\pm} | I \rangle$  between a final (F) state with isospin projection  $T_0^F$  and an initial (I) states with  $T_0^I$  in a decay generated by the raising (for  $\beta^-$  decay) and lowering ( $\beta^+$ ) isospin transition operator<sup>4</sup>  $\sqrt{2\Omega} T_{\pm}$ , which in the framework of our model is given as

$$|M_F|^2 = 2\Omega |\langle F; n(\tilde{T})T_0 \pm 1 | T_{\pm} | I; n(\tilde{T})T_0 \rangle|^2, \quad (9)$$

where  $|n(\tilde{T})T_0\rangle$  are the eigenvectors of the total Hamiltonian (6) with an almost good isospin  $\tilde{T}$  quantum number. Typically, the isospin impurity caused

<sup>4</sup>The factor of  $2\Omega$  appears due to the normalization of the basis operators adopted in the  $\text{sp}(4)$  algebraic model.

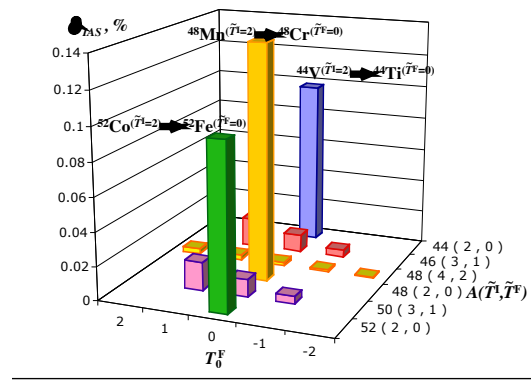


Figure 1:  $\text{Sp}(4)$  model estimate for the  $\delta_{IAS}$  isospin mixing correction [%] (10) to Fermi  $\beta$ -decay transition matrix elements between isobaric analog  $0^+$  states of almost good isospin  $\tilde{T}$  for the nuclei in the  $1f_{7/2}$  level ( $F/\Omega = 0.072$ ).

by isospin non-conserving forces in nuclei is estimated as a correction to the Fermi matrix element  $|M_F|^2$  of the superallowed  $\tilde{T}$  analog  $0^+ \rightarrow 0^+$  transition,  $\delta_C = 1 - |M_F|^2 / \{ \tilde{T}(\tilde{T} + 1) - T_0^F T_0^I \}$ . For more than two-state mixing, the degree of isospin admixture between isobaric analog  $0^+$  states should be estimated using the normalized transition matrix element between non-analog (NA) states (e.g. [37]),

$$\delta_{IAS} = \frac{|M_F^{NA}|^2}{\{ \tilde{T}(\tilde{T} + 1) - T_0^F T_0^I \}}, \quad (10)$$

where  $\tilde{T}$  is the almost good isospin of the parent nucleus (Figure 1 for the  $1f_{7/2}$  level). The small mixing of the  $0^+$  isospin eigenstates from different isospin multiplets reflects very small but nonzero  $|M_F^{NA}|^2$  matrix elements for non-analog  $\beta^\pm$  decay transitions (Figure 1 and first column of Table 2). In short, the theoretical  $\text{Sp}(4)$  model suggests the possible existence, albeit highly hindered, of  $\Delta T = 2$  non-analog  $\beta$ -decay transitions.

In general, the  $\delta_{IAS}$  correction may be very differ-

ent than the order of the  $\delta_{\tilde{T},T}$  overlap quantity

$$\delta_{\tilde{T},T} = \left| \left\langle n, T, T_0 | n(\tilde{T}) T_0 \right\rangle \right|^2 * 100[\%] \quad (11)$$

of the  $|n(\tilde{T}) T_0\rangle$  nuclear states with the isospin eigenvectors (Figure 2). This is because in decays the degrees of isospin mixing between non-analog states within both the parent and daughter nuclei are significant. As it is expected, the  $\delta_{\tilde{T},T}$  isospin mixing increases as  $Z$  and  $N$  approach one another and towards the middle of the shell. Although the isospin admixture is negligible for light nuclei in the  $j = 3/2$  orbit ( $\delta_{\tilde{T}=0,T=2} = 0.0001$  for  $^{36}\text{Ar}$ ,  $F/\Omega = 0.007$ ), it is clearly bigger for the  $j = 7/2$  level (Figure 2), yet less than 0.17%.

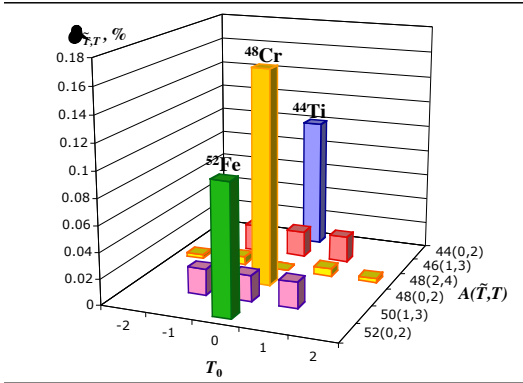


Figure 2: Sp(4) model estimate for the mixing overlap [%] of  $0^+$  states under consideration of almost good isospin  $\tilde{T}$  with the states of definite isospin for the nuclei in the  $1f_{7/2}$  level ( $F/\Omega = 0.072$ ).

The analysis of the results for the  $1f_{7/2}$  orbit shows that the mixing between isobaric analog  $0^+$  states (which is at least  $\Delta T = 2$  mixing) is on average 0.006% excluding even-even  $N = Z$  nuclei (Figure 1). This is on the order of a magnitude less than the mixing of the first excited  $0^+$  non-analog state due to isospin non-conserving interaction, which is typically about 0.04% for the  $1f_{7/2}$  level [23, 37]. In addition, it is smaller than possible Gamow-Teller transitions,

$< 0.02\%$  for the nuclei in the  $1f_{7/2}$  shell [23], that are found substantially larger with increasing mass number  $A$  [49, 16, 27]. This makes  $\delta_{IAS}$  mixing very difficult to be detected especially when the isospin-symmetry breaking correction ( $\delta_C$ ) to analog Fermi matrix elements in this level is on the order of a percent [11, 16].

Despite the lack of experimental data, a rough estimate for the order of the  $\delta_{IAS}$  mixing induced by  $H_{IM}$  (5) in the Sp(4) model can be obtained in comparison to other types of mixing corrections that are measured or calculated. For example, the  $\beta$ -decay transition from the ground state of  $^{46}\text{V}$  to the first excited  $0^+$  non-analog state in  $^{46}\text{Ti}$  at  $E_{0_1^+} = 2.61$  MeV yields an experimental correction of 0.053% [23], which is also reproduced by theoretical calculations [37]. The first excited  $0^+$   $\tilde{T} = 2$  state in  $^{46}\text{Ti}$  that is an isobaric analog to the ground state of  $^{46}\text{Ca}$  lies at  $E_{\tilde{T}=2} = 13.36$  MeV (as predicted by the Sp(4) model [32]) and its mixing into the  $\tilde{T} = 0$  ground state should yield an isospin symmetry-breaking correction on the order of:

$$\delta_{IAS} \sim \left( \frac{E_{0_1^+}}{E_{\tilde{T}=2}} \right)^2 0.053\% \sim 0.0020\%, \quad (12)$$

which is about the values the Sp(4) model yields for  $\Delta T = 2$  non-analog transition between  $^{46}\text{V}$  and  $^{46}\text{Ti}$  (Figure 1). Clearly, such a comparison is approximate with respect to the high accuracy of the isospin mixing effects. Yet the results show consistency with other theoretical calculations and are found not to contradict reasonable limits set up by experimental evidences.

Not surprising, the largest values for the  $\delta_{IAS}$  correction are observed for  $\Delta T = 2$   $\beta^\pm$ -decays to energetically accessible  $0^+$  ground states of even-even  $N = Z$  nuclei (Figure 1). While for these decays  $\delta_{IAS}$  is extremely small, namely less than 0.14%, as expected for the contribution of the higher-lying  $0^+$  states [37], it is comparable to the order of isospin-symmetry breaking corrections for the  $1f_{7/2}$  orbit that are typically taken into account [37]. The reason is that for the even-even  $N = Z$  nuclei the second-lying isobaric analog  $0^+$  states are situated relatively low due to a significant  $pn$  interaction (Figure 1). As



an example, one finds that for the decay to the  $^{48}\text{Cr}$  ground state  $\delta_{IAS}$  may be only about 5 times smaller [proportional to the ratio in the energies squared as in (12)] than an average  $\Delta T = 1$  isospin-symmetry breaking correction in the  $1f_{7/2}$  orbit and takes the latter to be around 0.6% [37]. Indeed, the  $\text{Sp}(4)$  model yields  $\delta_{IAS} = 0.14328\%$  for  $^{48}_{25}\text{Mn}^{(2)} \rightarrow ^{48}_{24}\text{Cr}^{(0)}$ .

Above all, the  $\delta_{IAS}$  results in Figure 1 clearly show the overall pattern and the order of significance of the isospin mixing under consideration. This is evident within the first-order approximation in terms of the  $F$  parameter ( $F \ll 1$ ) of  $\delta_{IAS}$  (Table 2), which for  $1f_{7/2}$  deviates on average by only 2% from its exact calculations in Figure 1. The  $\delta_{IAS}^{(1)}$  isospin mixing correction is then proportional to  $F^2$  and one finds out that its order of magnitude remains the same for large variations of the  $F$  parameter of more than 60%. In addition, greater  $F$  values are not very likely because the  $\delta_{IAS}$  estimates (Figure 1) fall close below an upper limit, which does not contradict experimental and theoretical results for other types of isospin mixing. It is worth mentioning that while the energies of the lowest isobaric analog  $0^+$  states determined directly the parameters of the model interaction, a quite good reproduction of the experimental higher-lying isobaric analog  $0^+$  state energies followed without any parameter adjustment [32]. This outcome is important because the energy difference between two isobaric analog  $0^+$  states within a nucleus directly affects the degree of their mixing.

Moreover, in this first-order approximation the ratio of any two isospin corrections within a shell, where the strengths of the effective interaction are assumed fixed, is independent of the parameters of the model interaction. This implies that such a ratio does not reflect at all the uncertainties of the interaction strength parameters but rather it is characteristic of the relative strength of both decays. We choose to compare  $\delta_{IAS}^{(1)}$  for different  $\Delta T = 2$   $\beta$ -decays to the isospin mixing correction, denoted by  $\delta_{IAS}^{(1)*}$ , of the decay between nuclear isobars with  $n = 4$  valence particles [such as the  $^{44}_{23}\text{V}^{(2)} \rightarrow ^{44}_{22}\text{Ti}^{(0)}$  decay for the  $1f_{7/2}$  orbit (Table 2)] due to the relative simplicity of these nuclear systems. The  $\delta_{IAS}^{(1)}/\delta_{IAS}^{(1)*}$  ra-

tio then identifies the decay, for which the maximum isospin mixing correction is expected in the  $1f_{7/2}$  orbit, namely  $^{48}_{25}\text{Mn}^{(2)} \rightarrow ^{48}_{24}\text{Cr}^{(0)}$ , and as well as the amount by which  $\delta_{IAS}$  of the other possible non-analog decays is relatively suppressed (Table 2). For example, the  $\delta_{IAS}$  correction for the  $^{44}_{23}\text{V}^{(2)} \rightarrow ^{44}_{22}\text{Ti}^{(0)}$  decay is around 1.5 times smaller than the maximum one and it is around 8 times smaller for the  $^{46}_{25}\text{Mn}^{(3)} \rightarrow ^{46}_{24}\text{Cr}^{(1)}$  decay. Such a ratio quantity exhibits a general trend of increasing  $\delta_{IAS}$  isospin mixing with  $Z$  within same isospin multiplets and as well it reveals enhanced  $\Delta T = 2$  decays to the ground state of even-even  $N = Z$  nuclei with increasing  $\delta_{IAS}$  towards the middle of the shell.

Furthermore, the ratio retains its behavior for the non-analog  $\beta$  decays between nuclei with the same valence proton and neutron numbers as in Table 2 but occupying the  $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$  major shell (Table 3). Therefore, among the non-analog  $\beta$  decays for the  $A = 60 - 64$  isobars with valence protons and neutrons in the  $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$  shell the  $\delta_{IAS}$  isospin mixing of the  $^{64}_{33}\text{As}^{(2)} \rightarrow ^{64}_{32}\text{Ge}^{(0)}$  decay is expected to be the largest with a tendency of a further increase towards the middle of the shell. While the decay mentioned above exhibits isospin mixing twice stronger than the one for the  $^{60}_{31}\text{Ga}^{(2)} \rightarrow ^{60}_{30}\text{Zn}^{(0)}$  decay, the other  $A = 60 - 64$   $\beta$  decays are up to 110 times slower (Table 3). In addition, one needs to calculate only the isospin mixing for the simplest case of four valence nucleons, for then the order of significance of  $\delta_{IAS}$  for the other  $A = 62 - 64$  decays follow directly from the estimations presented in Table 3.

Even though the strength of the isospin mixing interaction may differ between different model spaces, the ratio,  $\delta_{IAS}^{(1)}/\delta_{IAS}^{(1)*}$ , turns out to be of the same order for both the  $1f_{7/2}$  level (Tables 2) and the upper  $fp$  shell (Table 3). In addition, the ratio strongly correlates for both shells retaining the same behavior as one goes from one model space to the other. In short, the significance of the isospin mixing caused by a charge dependent  $J = 0$  pairing correlations is evident from Table 2 for the  $1f_{7/2}$  level and continues the same trend for the upper  $fp$  shell (Table 3).

Indeed, the relative strength of the first-order isospin mixing correction for non-analog Fermi  $\beta$ -

decays under consideration varies smoothly with the size of the model space (Figure 3). All the  $\Delta T = 2$  decays relative to the one with four valence nucleons need an isospin mixing correction to the transition matrix elements that increases with the occupation space. Such an increase however keeps all of the decays under consideration suppressed relatively to the  $n = 4$  decay (with the largest  $\delta_{IAS}^{(1)}$  still about four times smaller than  $\delta_{IAS}^{(1)*}$ ). An exception is the  $\Delta T = 2$  decay to the ground state of the  $N = Z$   $n = 8$  nucleus, which is significantly faster for any  $\Omega$  space size. This decay becomes 2.5 times faster in the pair-boson limit of very large  $\Omega$ , which is an increase of 77.6% compared to  $\Omega = 4$ . While the  $n = 6$  nuclear systems exhibit an increase of only 43.8%, the isospin mixing correction increases 1.8 times for all the  $n = 8$  decays to a daughter nucleus of almost good isospin  $\tilde{T} = 2$ . In short, the Sp(4) model allows one to easily estimate the relative strength of different decays within a shell and hence to identify the fastest decay as well as the ones that can be readily neglected in precise isospin mixing calculations.

### 3 Conclusions

Isospin mixing induced by a short-range charge dependent nuclear interaction is described microscopically within the framework of a group-theoretical approach based on the Sp(4) dynamical symmetry. The Sp(4) model interaction incorporates the main driving forces, including  $J = 0$  pairing correlations and implicit quadrupole-quadrupole term, that shape the nuclear pairing-governed isobaric analog  $0^+$  states in the  $1f_{7/2}$  level where the Sp(4) Hamiltonian correlates strongly with realistic interactions.

Empirical evidence such as scattering analysis and the coefficient related to the isotensor part of a general non-conserving force,  $c$ , reveals the charge dependence of the  $J = 0$  pairing correlations. Indeed, the slightly stronger proton-neutron pairing interaction than the like-particle (proton-proton or neutron-neutron) pairing interaction came out of the Sp(4) analysis in a quite good reproduction of the energies of the lowest isobaric analog  $0^+$  states and the  $c$ -coefficient for a wide-range nuclear systems. The

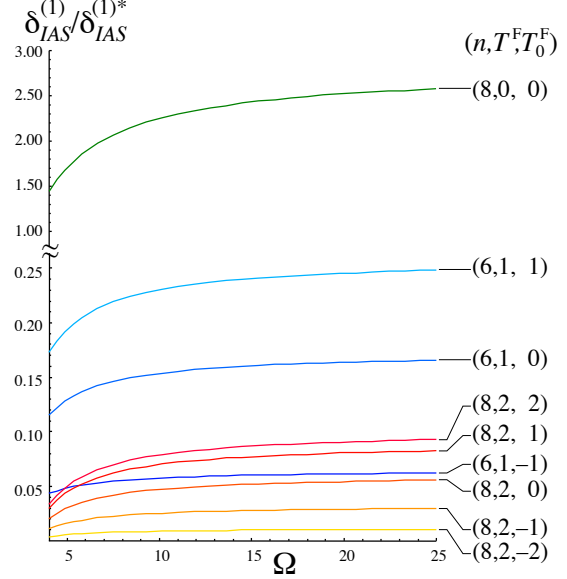


Figure 3: Sp(4) model estimate for the first-order mixing correction for the  $\Delta T = 2$  non-analog  $\beta$ -decays between  $0^+$  states under consideration for  $n$  valence nucleons occupying a model space of size  $\Omega$  (e.g.,  $\Omega = 4$  for  $1f_{7/2}$  and  $\Omega = 11$  for  $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ ) relative to the simplest  $n = 4$   $\Delta T = 2$  decay to the ground states of the  $N = Z$  nucleus.

freedom allowed in the algebraic model by introducing additional non-conserving forces reflects the symmetries observed in light nuclei (good isospin) and the comparatively larger symmetry-breaking as expected in medium-mass nuclei. The isospin-symmetry breaking due to coupling of isobaric analog  $0^+$  states in nuclei was estimated to be extremely small for nuclei in the  $1d_{3/2}$  and  $1f_{7/2}$  orbitals. However, the  $N = Z$  even-even light and medium mass nuclei are an exception. For these nuclei, strong pairing correlations, including a significant  $pn$  interaction, are responsible for the existence of comparatively larger isospin mixing, although the latter is still at least an order of a magnitude smaller than the overall isospin admixture in the ground state. The results also show



that a variation of more than 60% in the  $F$  isospin mixing parameter is required to reduce the present  $\delta_{IAS}$  results by an order of a magnitude.

The analysis also shows that there is a trend of increasing isospin mixing between isobaric analog  $0^+$  states due to a charge dependent  $J = 0$  pairing interaction towards the middle of the shell and for  $\Delta T = 2$  decays to the ground state of an even-even  $N = Z$  daughter nucleus. Such behavior is free of the uncertainties in the strength parameters of the interaction and is adequate for larger multi- $j$  shell domains such as  $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ . For nuclei with valence protons and neutrons occupying the  $1f_{7/2}$  level the strongest non-analog decay is identified to be  $^{48}_{25}\text{Mn}^{(2)} \rightarrow ^{48}_{24}\text{Cr}^{(0)}$  with the  $\delta_{IAS}$  isospin mixing correction being 1.5 to 300 times smaller for the rest of the decays. In the upper  $fp$  shell among the decays between nuclei with mass  $A = 60 - 64$  the  $^{64}_{33}\text{As}^{(2)} \rightarrow ^{64}_{32}\text{Ge}^{(0)}$  decays is found to be the fastest while the isospin mixing decreases for the other decays 2 to 250 times.

In general, relative to the simplest decay between isobars with four valence nucleons, the parameter-free isospin mixing corrections for the same number of valence nucleons increase with the size of the occupation space. Hence, such a ratio is larger in the  $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$  major shell than in the  $1f_{7/2}$  single- $j$  orbit. The trend observed allows for estimates for the isospin mixing correction to  $\beta$ -decay matrix elements between isobars with four to eight valence particles but filling other major shells. Moreover, the present study provides for the order of significance of isospin mixing due to pairing correlations for the decays under consideration if only the  $\delta_{IAS}$  mixing correction for the simplest decay is provided by model calculations or experimental observations. Hence large and negligible isospin mixing corrections are easily identified. For example, in the  $1f_{7/2}$  orbit the  $\text{Sp}(4)$  model yields  $\delta_{IAS} = 0.098\%$  for  $^{44}_{23}\text{V}^{(2)} \rightarrow ^{44}_{22}\text{Ti}^{(0)}$  and hence for the fastest  $^{48}_{25}\text{Mn}^{(2)} \rightarrow ^{48}_{24}\text{Cr}^{(0)}$  decay it is  $\delta_{IAS} = 0.143\%$ .

In short, the charge dependence of the nuclear force, being a very challenging problem, yields results, based on a simple group-theoretical approach, that are qualitatively as well as quantitatively con-

sistent with the observations. The  $\text{sp}(4)$  algebraic model yields an estimate for the decay rates of possible non-analog  $\beta$ -decay transitions due to a pure strong interaction, which, though few of them may affect slightly precise calculations, are not expected to comprise the dominant contribution to the isospin-symmetry breaking correction tested in studies of superallowed Fermi  $\beta$ -decay transitions.

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Table 2: Non-analog  $\beta$ -decay transitions to energetically accessible  $0^+$  states under consideration for nuclei in the  $1f_{7/2}$  level along with the parameter-free ratio of the first-order isospin mixing  $\delta_{IAS}^{(1)}$  relative to  $\delta_{IAS}^{(1)}$  of the  ${}^{44}_{23}\text{V}^{(2)} \rightarrow {}^{44}_{22}\text{Ti}^{(0)}$  decay (denoted as  $\delta_{IAS}^{(1)*}$ ) in the framework of the Sp(4) model. (There are no available experimental values for comparison.)

$\beta$ -decay		$\frac{\delta_{IAS}^{(1)}}{\delta_{IAS}^{(1)*}}$
${}^A_Z X^{(\tilde{T}_X)} \rightarrow {}^A_{Z-1} Y^{(\tilde{T}_Y)}$		
${}^{44}_{23}\text{V}^{(2)} \rightarrow {}^{44}_{22}\text{Ti}^{(0)}$		1.000
${}^{46}_{25}\text{Mn}^{(3)} \rightarrow {}^{46}_{24}\text{Cr}^{(1)}$		0.173
${}^{46}_{24}\text{Cr}^{(3)} \rightarrow {}^{46}_{23}\text{V}^{(1)}$		0.115
${}^{46}_{23}\text{V}^{(3)} \rightarrow {}^{46}_{22}\text{Ti}^{(1)}$		0.043
${}^{48}_{27}\text{Co}^{(4)} \rightarrow {}^{48}_{26}\text{Fe}^{(2)}$		0.034
${}^{48}_{26}\text{Fe}^{(4)} \rightarrow {}^{48}_{25}\text{Mn}^{(2)}$		0.030
${}^{48}_{25}\text{Mn}^{(4)} \rightarrow {}^{48}_{24}\text{Cr}^{(2)}$		0.020
${}^{48}_{24}\text{Cr}^{(4)} \rightarrow {}^{48}_{23}\text{V}^{(2)}$		0.011
${}^{48}_{23}\text{V}^{(4)} \rightarrow {}^{48}_{22}\text{Ti}^{(2)}$		0.004
${}^{48}_{25}\text{Mn}^{(2)} \rightarrow {}^{48}_{24}\text{Cr}^{(0)}$		1.452
${}^{50}_{27}\text{Co}^{(3)} \rightarrow {}^{50}_{26}\text{Fe}^{(1)}$		0.173
${}^{50}_{26}\text{Fe}^{(3)} \rightarrow {}^{50}_{25}\text{Mn}^{(1)}$		0.115
${}^{50}_{25}\text{Mn}^{(3)} \rightarrow {}^{50}_{24}\text{Cr}^{(1)}$		0.043
${}^{52}_{27}\text{Co}^{(2)} \rightarrow {}^{52}_{26}\text{Fe}^{(0)}$		1.000

Table 3: Non-analog  $\beta$ -decay transitions to energetically accessible  $0^+$  states under consideration for  $A = 60-64$  nuclei in the the upper  $fp$  shell along with the parameter-free ratio of the first-order isospin mixing  $\delta_{IAS}^{(1)}$  relative to  $\delta_{IAS}^{(1)}$  of the  ${}^{60}_{31}\text{Ga}^{(2)} \rightarrow {}^{60}_{30}\text{Zn}^{(0)}$  decay (denoted as  $\delta_{IAS}^{(1*)}$ ) in the framework of the  $\text{Sp}(4)$  model. (There are no available experimental values for comparison.)

$\beta$ -decay		$\frac{\delta_{IAS}^{(1)}}{\delta_{IAS}^{(1*)}}$
${}^A_Z X^{(\tilde{T}_X)} \rightarrow {}^A_{Z-1} Y^{(\tilde{T}_Y)}$		
${}^{60}_{31}\text{Ga}^{(2)} \rightarrow {}^{60}_{30}\text{Zn}^{(0)}$		1.000
${}^{62}_{33}\text{As}^{(3)} \rightarrow {}^{62}_{32}\text{Ge}^{(1)}$		0.233
${}^{62}_{32}\text{Ge}^{(3)} \rightarrow {}^{62}_{31}\text{Ga}^{(1)}$		0.156
${}^{62}_{31}\text{Ga}^{(3)} \rightarrow {}^{62}_{30}\text{Zn}^{(1)}$		0.058
${}^{64}_{35}\text{Br}^{(4)} \rightarrow {}^{64}_{34}\text{Se}^{(2)}$		0.081
${}^{64}_{34}\text{Se}^{(4)} \rightarrow {}^{64}_{33}\text{As}^{(2)}$		0.072
${}^{64}_{33}\text{As}^{(4)} \rightarrow {}^{64}_{32}\text{Ge}^{(2)}$		0.049
${}^{64}_{32}\text{Ge}^{(4)} \rightarrow {}^{64}_{31}\text{Ga}^{(2)}$		0.026
${}^{64}_{31}\text{Ga}^{(4)} \rightarrow {}^{64}_{30}\text{Zn}^{(2)}$		0.009
${}^{64}_{33}\text{As}^{(2)} \rightarrow {}^{64}_{32}\text{Ge}^{(0)}$		2.301
$\vdots$		